

**APJ ABDULKALAM TECHNOLOGICAL UNIVERSITY**  
**08 PALAKKAD CLUSTER**

Q. P. Code : CS-1D-18-2

(Pages: 4)

Name: .....

Reg. No:.....

**FIRST SEMESTER M.TECH. DEGREE EXAMINATION DEC 2018**

Branch: Computer Science and Engineering Specialization: Computer Science and Engineering

**08 CS 6041 MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE**

Time:3 hours

Max.marks: 60

Answer all six questions.

Modules 1 to 6:Part 'a' of each question is compulsory and answer either part 'b' or part 'c' of each question.

Q.no.	Module 1	Marks
1.a	For a given matrix A of order 3, $ A  = 32$ and two of its eigen values are 8 and 2. Find the trace of A.	3
	Answer b or c	
		6
b	Using Cayley Hamilton theorem, find $A^{-1}$ and $A^4$ where $A = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$	
c	Find the singular value decomposition of the matrix $A = \begin{bmatrix} 5 & 5 \\ -1 & 7 \end{bmatrix}$	6
Q.no.	Module 2	Marks
2.a	A family has 2 children. What is the probability that both are boys given that at least one of them is boy? Assume that each of the outcomes are equally likely.	3
	Answer b or c	
b	For a certain binary communication channel, the probability that a transmitted '0' is received as '0' is 0.95 and the probability that a transmitted '1' is received as '1' is 0.90. If the probability that a '0' is transmitted is 0.4, find the probability that (i) a '1' is received and (ii) a '1' was transmitted given that a '1' was received.	6

- c Consider 2 urns where 1<sup>st</sup> contains 2 white and 7 blue balls and the second contains 5 white and 6 blue balls. We flip a fair coin and then draw a ball from the 1<sup>st</sup> urn or 2<sup>nd</sup> urn depending on whether the outcome of the toss was head or tail respectively. What is the conditional probability that the outcome of the toss was head given that a white ball was selected? **6**

<b>Q.no.</b>	<b>Module 3</b>	<b>Marks</b>
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- 3.a** A man is at an integral point on the x-axis between the origin and the point 3. He takes a unit step to the right with probability 1/3 or to the left with probability 2/3, unless he is at the origin, where he takes a step to the right to reach the point 1 or is at the point 3, where he takes a step to the left to reach the point 2. Write down the transition probability matrix. **3**

**Answer b or c**

- b** The TPM of a markov chain  $X_n, n \geq 0$  having three states 1,2 and 3 is **6**  

$$P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.6 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{bmatrix}$$
 and the initial probability distribution is  $P(0) = [0.5 \ 0.3 \ 0.2]$ . Find (a)  $P[X_2 = 2]$  (b)  $P[X_3 = 3, X_2 = 2, X_1 = 1, X_0 = 3]$
- c** A salesman's territory consists of 3 cities A,B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in B. However, if he sells either in B or C, then the next day he is twice as likely to sell in city A as in the other city. How often does he sell in each of the cities in the steady state? **6**

<b>Q.no.</b>	<b>Module 4</b>	<b>Marks</b>
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- 4.a** A fisherman catches fish at a poisson process rate of 2 per hour from a large pond with lots of fish. If he starts fishing at 10am, what is the probability that he catches 1 fish by 10.30am and 3 fish by noon (12pm)? **3**

**Answer b or c**

- b** On the average, a submarine on patrol sights 6 enemy ships per hour. Assuming that the number of ships sighted in a given length of time is a poisson variate, find the probability of sighting **6**
- i) 6 ships in the next half-an-hour
- ii) 4 ships in the next 2h
- iii) at least 1 ship in the next 15 min
- iv) at least 2 ships in the next 20 min.

- c If people arrive at a bookstall with a poisson process with a mean rate of 3 per minute. Find the probability that the interval between two consecutive arrivals is
- (i) more than 2min
  - (ii) between 2min and 3min
  - (iii) 4min or less

<b>Q.no.</b>	<b>Module 5</b>	<b>Marks</b>
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| <b>5.a</b> | Consider an M/M/1 queueing system with $\lambda = 6$ and $\mu = 8$ , find the probability of at least 10 customers in the system. | <b>4</b> |
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**Answer b or c**

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| <b>b</b> | Customers arrive at a one-man barber shop according to Poisson process with a mean inter arrival time rate of 12min. Customers spend an average of 10min in the barber's chair. | <b>8</b> |
|----------|---|----------|

- (i) What is the expected number of customers in the barber shop and in the queue?
- (ii) How much time can a customer expect to spend in the barber's shop?
- (iii) What is the probability that the waiting time in the system is greater than 30min?
- (iv) Management will provide another chair and hire another barber, when a customer's waiting time in the shop exceeds 1.25hour. How much must the average rate of arrivals increase to warrant a second barber?

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| <b>c</b> | A supermarket has two girls attending to sales at the counters. If the service time for each customer is exponential with mean 4min and if people arrive in Poisson fashion at the rate of 10 per hour, | <b>8</b> |
|----------|---|----------|

- (i)What is the probability that a customer has to wait for service?
- (ii)What is expected percentage of idle time for each girl?
- (iii)If the customer has to wait in the queue, what is the expected length of his waiting time?

<b>Q.no.</b>	<b>Module 6</b>	<b>Marks</b>
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| <b>6.a</b> | Write down Pollaczek-Khinchine formula. | <b>4</b> |
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**Answer b or c**

- b** A car manufacturing plant uses one big crane for loading cars into a truck. Cars arrive for loading by the crane according to a Poisson distribution with a mean of 5 cars per hour. Given that the service time for all cars is constant and equal to 6 minutes, determine  $L_s$ ,  $L_q$ ,  $W_s$  and  $W_q$ . **8**
- c** A one man barber shop takes exactly 25 minutes to complete one hair-cut. If customers arrive at the barber shop in a Poisson fashion at an average rate of one every 40 minutes, how long on the average a customer spends in the shop? Also find the average time a customer must wait for service. **8**